Studying Social Inequality with Data Science

INFO 3370 / 5371
Spring 2023

Causal Assumptions

Learning goals for today

By the end of class, you will be able to

- Formalize causal assumptions in Directed Acyclic Graphs (DAGs)
- Use DAGs to find a sufficient adjustment set of variables within which a statistical association is causal

What is a **Directed Acyclic Graph (DAG)**?

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- Acyclic: There are no cycles

Not DAGs:

$$\begin{tikzcd} & B \arrow[dr, dash] & \\ A \arrow[ur, dash] \arrow[rr, dash] & & C \end{tikzcd}$$

Undirected

$$\begin{tikzcd} & B \arrow[dr] & \\ A \arrow[ur] & & C \arrow[ll] \end{tikzcd}$$

Cyclic

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Causal assumptions become

- visually intuitive
- mathematically precise

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$$\text{College} \longrightarrow \text{Earnings}$$

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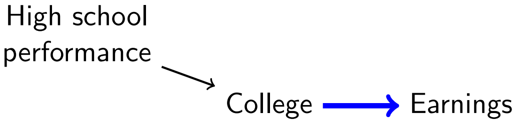

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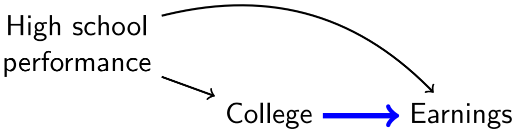

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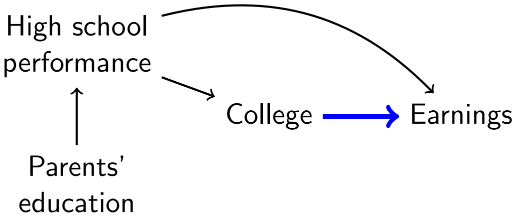

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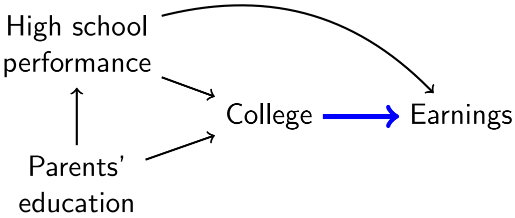

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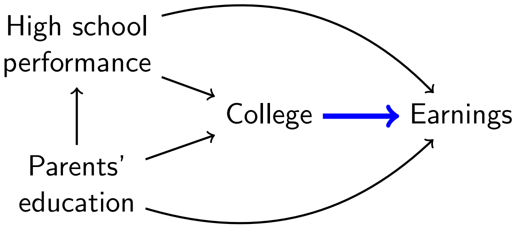

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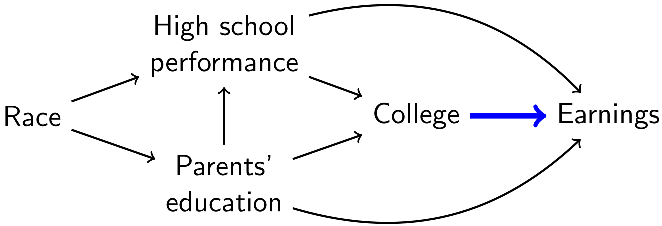

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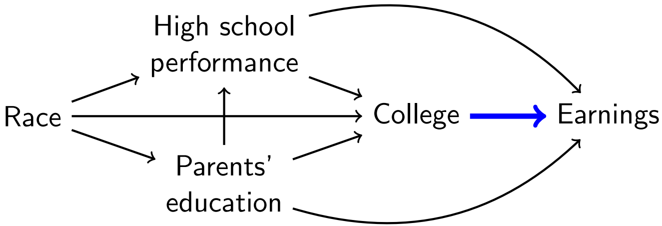

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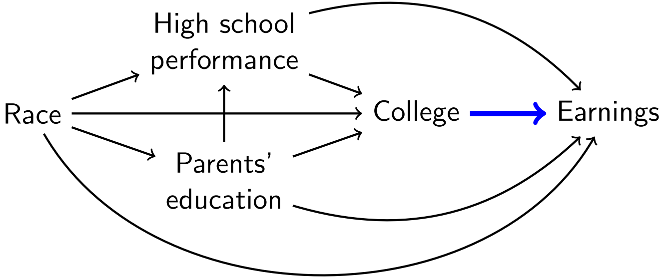

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$$A \longrightarrow Y$$

You finished college — Your pay

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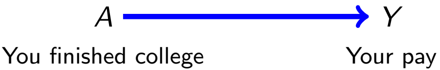

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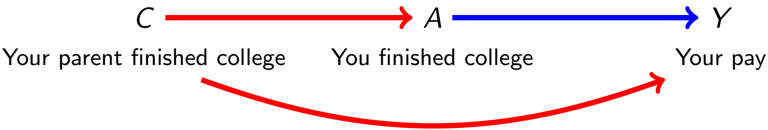

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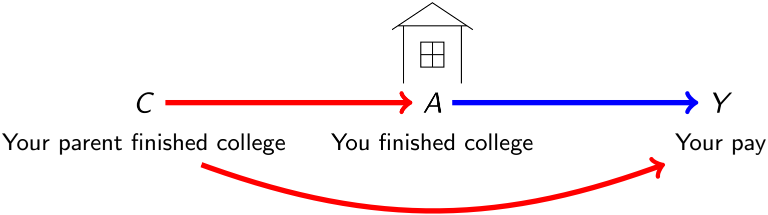

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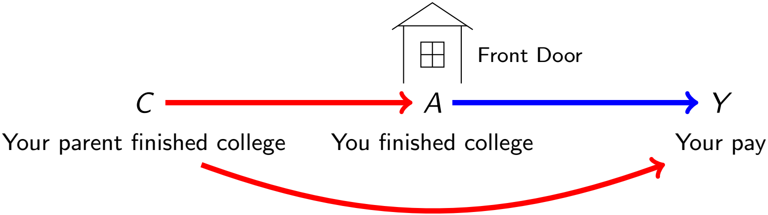

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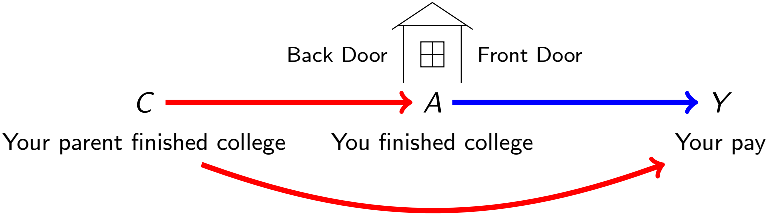

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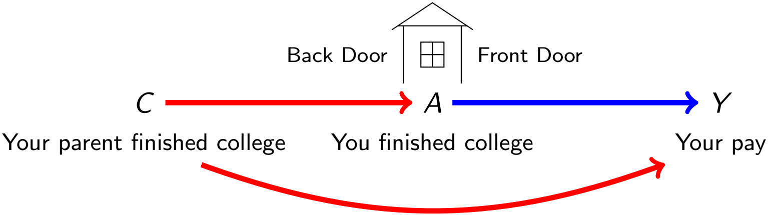

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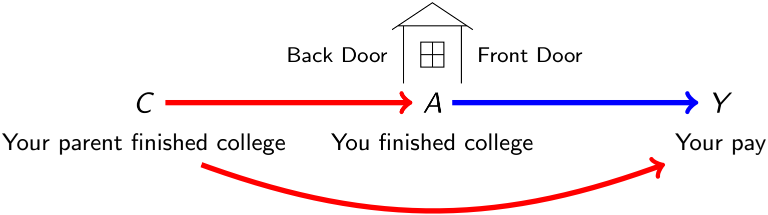

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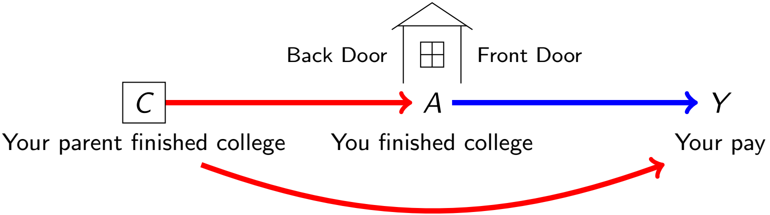

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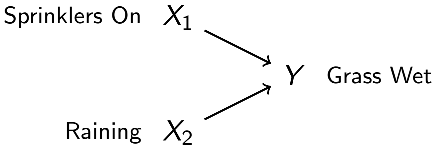

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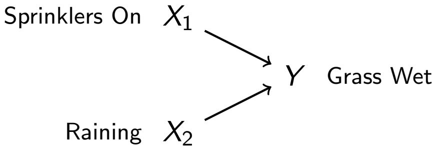

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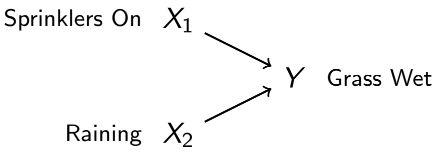

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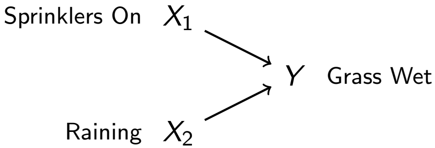

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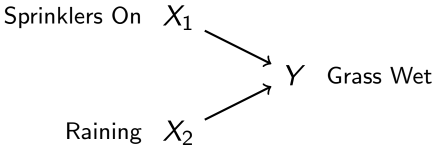

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- X_1 is independent of X_2
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- Conditioning on Y opens the path
  - If the grass is wet (conditional on $Y = 1$), then either (Sprinklers On) or (Raining)

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Ancestors vs. Colliders

Conditioning on an ancestor **closes** an open path

$$\begin{tikzcd} \boxed{X} \arrow[r] \arrow[rr, bend right] & A & Y \end{tikzcd}$$

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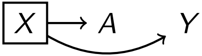

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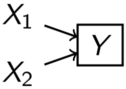

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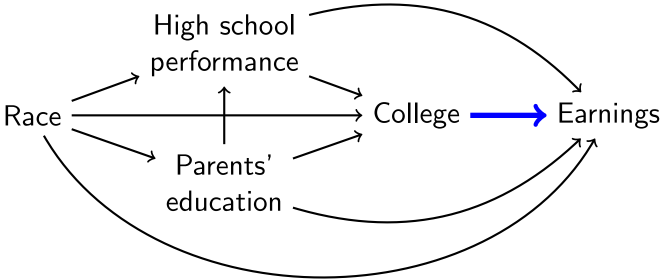
High school performance
Race
College
Earnings
Parents' education

How to find adjustment variables to identify causal effects

Goal:
Block all backdoor paths so treatment A and outcome Y are associated only by the causal path

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Backdoor path: Any sequence of edges $A \leftarrow \texttt{nodes} \rightarrow Y$

Blocked if it contains an adjusted variable along a fork

$$A \leftarrow \boxed{C} \rightarrow Y$$
$$A \leftarrow \boxed{C} \leftarrow \cdots \rightarrow Y$$
$$A \leftarrow \cdots \rightarrow \boxed{C} \rightarrow Y$$

Blocked if it contains an unadjusted collider

$$A \rightarrow C \leftarrow Y$$

Exercise 1

Find adjustment sets that identify the effect of A on Y

$$
\begin{tikzcd}
X_1 \arrow[r] \arrow[d] & A \arrow[r] & Y \\
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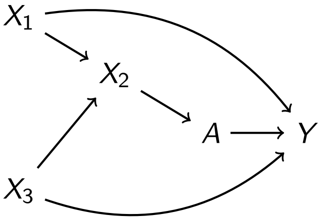
- Condition on X_3: $A \leftarrow X_1 \rightarrow X_2 \rightarrow \boxed{X_3} \rightarrow Y$
- Any combination of the above

Exercise 2

Find 3 sufficient adjustment sets to identify $A \to Y$



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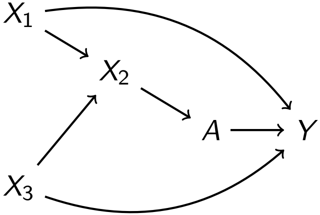

Answer: $\{X_2\}, \{X_1, X_3\}, \{X_1, X_2, X_3\}$

Exercise 3

What is the smallest adjustment set that identifies $A \rightarrow Y$?

$$
\begin{tikzcd}
X_1 \arrow[dr] \arrow[rrrdd, bend left] & & & \\
& X_2 & & \\
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Answer: The empty set! Don't condition on anything.
The collider X_2 already blocks the path.

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- Use DAGs to find a sufficient adjustment set of variables within which a statistical association is causal